Determination of the static torsion angle using dynamic measurements of the car body: FRF vs modal approach

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Abstract
The static torsion angle of a car body is an important design parameter to measure the global resistance it offers to twisting. This paper deals with two methods to derive global static stiffness from dynamic measurements: a FRF based technique which uses the concept of eRCF introduced by Pasha et al. and a special writing of the modal approach. This proposed writing includes lower and upper residual terms which are required to get reasonable accuracy in the stiffness estimates. The theories of the two methods are extended to be able to compute the static torsion angle from degrees of freedom which are different of the ones used for loadings and boundary conditions. This extension is needed to compare stiffness estimates with measured values from test rigs. An experimental case involving dynamic measurements of a car body is presented. The static torsion angle is estimated with the two methods and compared to a reference value measured for the same car body using a static test facility.

1 Introduction

The main industrial objective of this work is to reduce the costs of the experiments which are needed to determine the global static stiffness of car bodies. Traditionally, dedicated test facilities are used to measure static torsion and bending angles. They require expensive instrumentation and intensive resources. The use of standard dynamic measurements equipments to derive global static stiffness from dynamic frequency response function measurements requires significantly less resources and expenses. Two methods are investigated to extract the static stiffness information from the measured dynamic data. The eRCF method deals with frequency response functions, whereas the modal method deals with intrinsic properties of the structure, i.e. its modes of vibration. After a short state of the art, the theories of both methods are presented in the particular case of the determination of the static torsion stiffness. In the modal approach, residual stiffness vectors are introduced in the expression of the static compliance as the cumulative contribution of the modes of vibration in order to avoid modal truncation. The study of the cumulative contribution of the modes on the static compliance should improve the understanding of the relation between statics and dynamics. The equations of the two methods are rewritten to be able to compute the static torsion angle from degrees of freedom which are different of the ones used for loadings and boundary conditions. This development is needed to compare stiffness estimates with reference values from static test facilities. An experimental application of the two methods is presented. Dynamic measurements are performed on a car body of a mass-produce vehicle. The measurement set-up and the modal extraction parameters are briefly discussed. The static torsion angle of the car body is also measured to have a reference value to be compared with the angles determined with the two methods. Finally, conclusions on the accuracy of the two methods and perspectives of the work are given.

2 Static torsion stiffness

2.1 State of the art

Most of the existing techniques are described in [1] in the context of static and dynamic car body stiffness. The figure 1 gives an overview of the global methodology described in the paper to determine the static stiffness from the compliance matrix measurement.

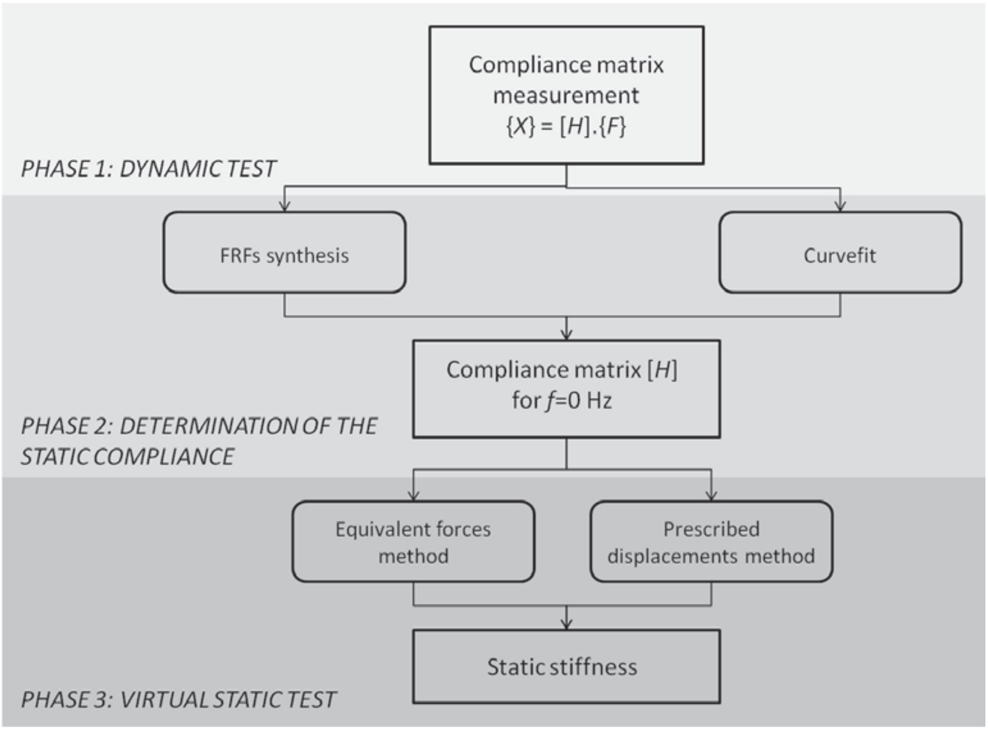

Figure 1: Global methodology to determine static stiffness from dynamic measurements

The compliance matrix is determined at 0 Hz by the synthesis of FRFs or by a simple curvefit of the compliance spectra measured at higher frequencies. Two methods are proposed to compute the static stiffness from the compliance matrix at 0 Hz. The equivalent forces method is discussed in the next section. The other method, namely the prescribed displacements method, is not discussed.

2.2 eRCF

The enhanced Rotational Compliance Function (eRCF) defined in [2] is a vectorization of the force equivalence method introduced in [3]. The force equivalence method is one of the two methods to perform a virtual static test from the compliance matrix at 0 Hz. It consists in the determination of a free-free configuration equivalent to the fixed-free configuration of the static torsion test as presented in the figure 2.

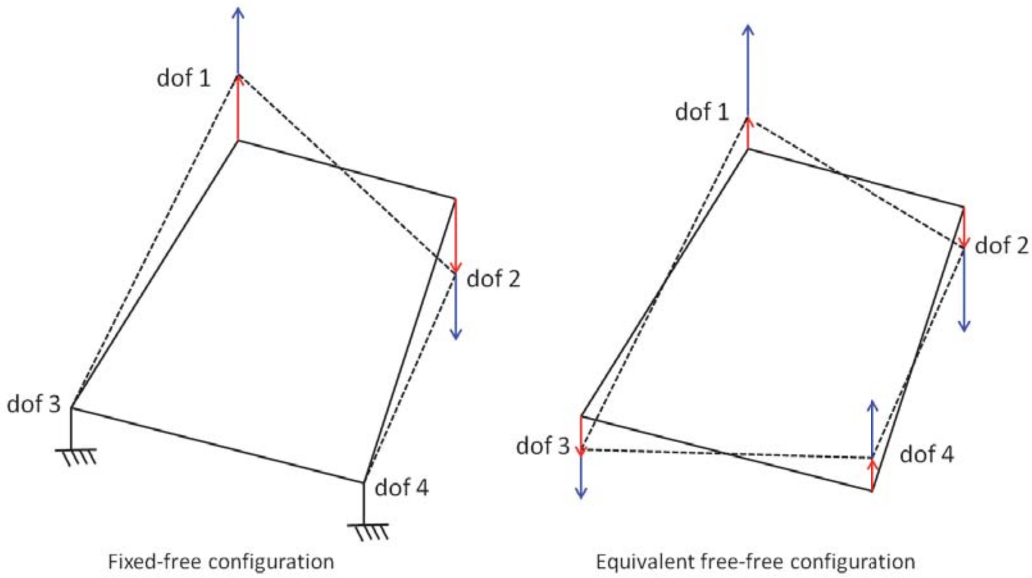

Figure 2: Equivalent configurations to estimate the static torsion stiffness

The idea is to calculate the deflections at the load application points as a function of the equivalent forces. The equivalent forces are a set of forces that is representative of the static test to be correlated with (torsion or bending). The eRCF is given as a function of the frequency by the relation:

$$eRCF(\omega_i) \Rightarrow \frac{\Delta\theta}{M} = \{V\}^T \left[H(\omega_i)\right] \{V\} \tag{1}$$

with $\Delta\theta$, the twist angle, M the torque and $\{V\}$ the moment scaling vector as:

$$\{V\}^T = \left\{ \frac{-1}{L_r} \quad \frac{1}{L_r} \quad \frac{-1}{L_f} \quad \frac{1}{L_f} \right\} \tag{2}$$

2.3 Modal approach

As explained in [4], the compliance matrix can be written in the modal domain in function of the rigid body modes, the flexible modes, the modal parameters (eigenvalues, modal viscous damping coefficient and generalized masses) and some residual terms ($G_{ss,res}$, $H_{ss,res}$):

$$G_{ss}(\omega) = \frac{A_{ss}}{-\omega^2} + \sum_k H_k(\omega) G_{ss,k} + G_{ss,res} + \omega^2 H_{ss,res} + ... \tag{3}$$

with A_{ss} the accelerances of the rigid body modes Φ_{rs} with the generalized masses m_{rr} (mass or principal inertia of the structure for mode r):

$$A_{ss} = \sum_r \frac{\Phi_{sr}\Phi_{rs}}{m_{rr}} \tag{4}$$

$H_k(\omega)$ the dynamic amplification factor of elastic mode k:

$$H_k(\omega) = \frac{1}{1 - \left(\frac{\omega}{\omega_k}\right)^2 + i2\zeta_k \frac{\omega}{\omega_k}} \tag{5}$$

$G_{ss,k}$ the effective compliance of elastic mode k:

$$G_{ss,k} = \frac{\Phi_{sk}\Phi_{ks}}{\omega_k^2 m_{kk}} \tag{6}$$

The determination of G_{ss} for $\omega = 0$ needs to compute $\frac{1}{-\omega^2} A_{ss}$. This term may tend to infinite. A solution to avoid this problem is to use the definition of the static compliance described by [5]. The static compliance is defined as the ratio of the generalized displacement D to the generalized force F at 0 Hz:

$$C = \frac{D}{F} = \frac{\sum_j \chi_j y_j}{F} \tag{7}$$

where y_j are the displacements at load and support points and χ_j, the load scale factors. The authors explain this definition automatically removes any rigid body deflections at the load points that result from the deflection of the supports. They develop for a single vibration mode the equation of the static compliance to a final form:

$$C_r = \frac{\left[\sum_i \chi_i \Phi_{ri}\right]^2}{\omega_r^2 m_r} \tag{8}$$

where Φ_{ri}, ω_r and m_r are respectively the r^{th} modal deflection at dof i, the r^{th} eigenvalue and the r^{th} modal mass. Since this is a quadratic form the compliance is positive as long as the mode shapes are real. For complex modes, [6] gives a new form for the static compliance:

$$C_r = \frac{1}{\omega_r^2 m_r}\left[A^2 - B^2 + \frac{2\zeta}{\sqrt{1-\zeta^2}} AB\right] \tag{9}$$

with:

- $A = \sum_i \chi_i \Re\left(\Psi_{ri}\right)$
- $B = \sum_i \chi_i \Im\left(\Psi_{ri}\right)$
- ζ_r the modal viscous damping factor of the r^{th} mode

The total compliance C is the sum of the compliances due to the individual real (or complex) modes:

$$C = \sum_r C_r = \sum_r \frac{\left[\sum_i \chi_i \Phi_{ri}\right]^2}{\omega_r^2 m_r} \tag{10}$$

This form is equivalent to the contribution of the elastic modes in the equation 3 with $H_{k,(\omega=0)} = 1$.

Because of the modal truncation, the total compliance equation can be completed with the contributions of the upper residual term as:

$$C = \sum_r \frac{\left[\sum_i \chi_i \Phi_{ri}\right]^2}{\omega_r^2 m_r} + \{V\}^T \left[H_{res}\right] \{V\} \tag{11}$$

where H_{res} is the upper residual term (residual stiffness) used to approximate modes at frequencies above ω_{max}. This residual term is determined by a modal extraction algorithm to fit the synthesized FRFs with the measured ones. This is assuming the data to have the dimension of displacement over force.

3 Adaption to the actual static stiffness set-up

3.1 New input/output degrees of freedom

The force equivalent method assumes the observation degrees of freedom and the excitation and boundary conditions degrees of freedom are the same. Taking into account the methodology of the static test facility leads to separate these degrees of freedom and to rewrite the equations of the both methods, i.e. the eRCF and the modal approach. Four degrees of freedom are added to the problem to be solved. As a result, the total number of degrees of freedom is eight, i.e. one degree of freedom in vertical translation for the eight points of interest. The vector of the degrees of freedom is then written as:

$$\{u\}^T = \{u_{FEL},\quad u_{FER},\quad u_{REL},\quad u_{RER},\quad u_{FRL},\quad u_{FRR},\quad u_{RRL},\quad u_{RRR}\} \tag{12}$$

With the following glossary as defined in figure 3:

- FEL: Front Excitation Left
- FER: Front Excitation Right
- FRL: Front Response Left
- FRR: Front Response Right
- REL: Rear Excitation Left
- RER: Rear Excitation Right
- RRL: Rear Response Left
- RRR: Rear Response Right

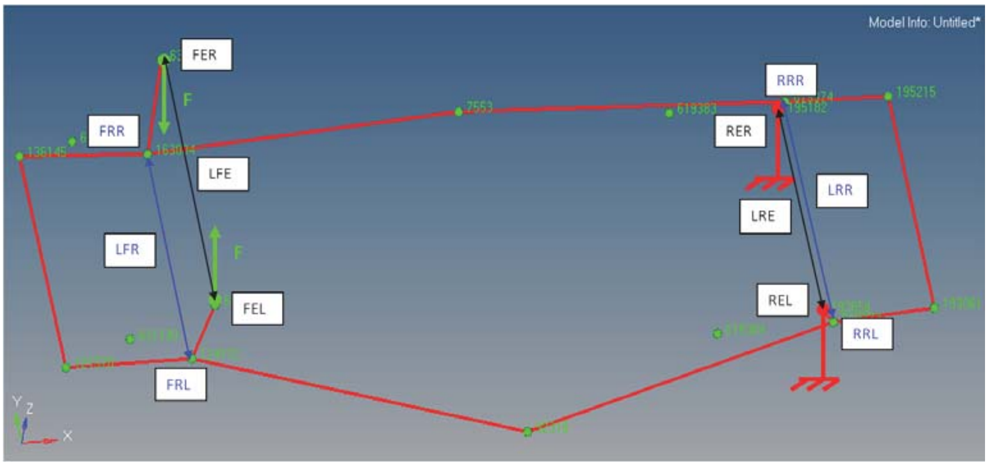

Figure 3: Definition of the excitation and response points of the car body

The glossary of the distances between the points is:

- LFE: Length Front Excitation (distance between FEL and FER)
- LFR: Length Front Response (distance between FRL and FRR)
- LRE: Length Rear Excitation (distance between REL and RER)
- LRR: Length Rear Response (distance between RRL and RRR)

3.2 eRCF with new IO DOFS

In the case of the eRCF, the Moment Scaling Vector V is used to take into account the torque and the clamping conditions. It is written with eight degrees of freedom as:

$$\{V\}^T = \left\{ \tfrac{1}{LFE} \quad \tfrac{-1}{LFE} \quad \tfrac{-1}{LRE} \quad \tfrac{1}{LRE} \quad 0 \quad 0 \quad 0 \quad 0 \right\} \tag{13}$$

The four last terms are zero, because no excitation and no boundary conditions are applied on the points where the static torsion angle is determined. The system to be solved is then:

$$\{u\} = M\,[H]\,\{V\} \tag{14}$$

Where M is the scalar value of the torque applied to the car body and $[H]$ the dynamic compliance matrix (displacement/force). In order to obtain the global torsion angle as:

$$\Delta\theta = \theta_{av} - \theta_{ar} \approx \frac{u_{FRL} - u_{FRR}}{LFR} - \frac{u_{RRL} - u_{RRR}}{LRR} \tag{15}$$

A new vector A, namely the Angle Computing Vector, is introduced to multiply the response vector u. Its expression is:

$$\{A\}^T = \left\{ 0 \quad 0 \quad 0 \quad 0 \quad \tfrac{1}{LFR} \quad \tfrac{-1}{LFR} \quad \tfrac{-1}{LRR} \quad \tfrac{1}{LRR} \right\} \tag{16}$$

Finally, the global torsion angle is obtained with the eRCF using the relation:

$$\Delta\theta_{(\omega)} = M \cdot eRCF(\omega) = M \cdot \{A\}^T\,[H(\omega)]\,\{V\} \tag{17}$$

3.3 Modal Method with new IO DOFS

The angle is frequency dependent, because it is computed from the matrix $[H]$. Its static value is estimated with a curve fitting of the eRCF function at 0 Hz.

In the case of the modal method, the Moment Scaling Vector is $\{V\} = \{\chi_i\}$ (with $i = 1, 8$) and the Angle Computing Vector is $\{A\} = \{\mu_j\}$ (with $j = 1, 8$)

According to the force equivalence method, the multiplication by the vector V allows the application of the torque and the boundary conditions. The multiplication by the vector A^T leads to compute the static torsion stiffness from the output degrees of freedom, i.e. the four new degrees of freedom introduced in the problem to be solved.

In the case of normal modes, the expression of the static compliance C becomes:

$$C = \sum_r \frac{\sum_i \sum_j \chi_i \mu_j \Phi_{ri} \Phi_{rj}}{\omega_r^2 m_r} + \{A\}^T\,[H_{res})]\,\{V\} \tag{18}$$

With:

- Φ_{ri} the i^{th} degree of freedom of the r^{th} mode
- Φ_{rj} the j^{th} degree of freedom of the r^{th} mode
- ω_r the natural frequency of the r^{th} mode

- m_r the modal mass of the r^{th} mode
- $[H_{res}]$ the matrix of the residual vectors determined at the points where the torque and the boundary conditions are applied

As an example, the cumulative static compliance is plotted in function of the frequency in figure 4:

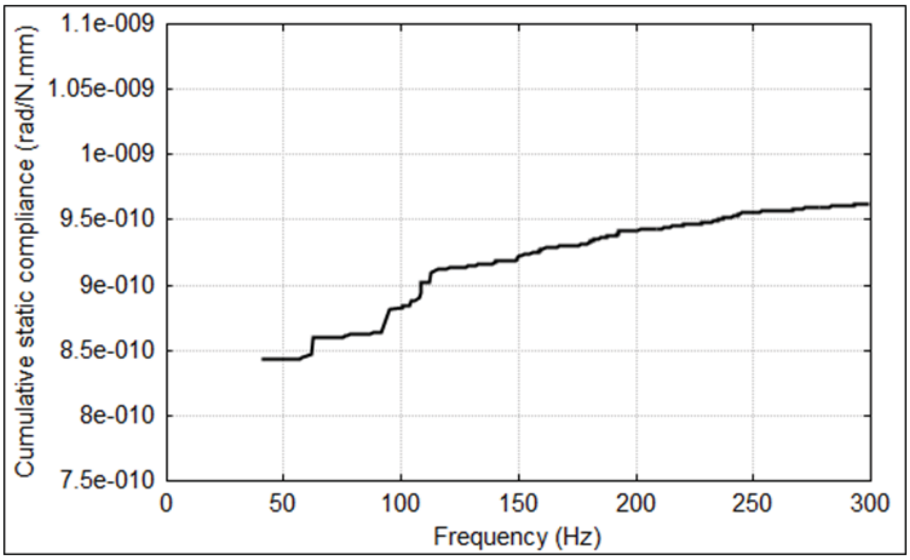

Figure 4: Cumulative static compliance

It is obtained as the cumulative sum of the contributions of the flexible modes of a car body to the static compliance. The modes are computed using a finite elements model of the body in white. The contribution of the first mode (the torsion mode) is 87.6% of the total static compliance computed from the contributions of all the flexible modes up to 300 Hz.

In the case of complex modes $\{\Psi_r\}$, the expression of the static compliance C becomes:

$$C = \sum_r \frac{1}{\omega_r^2 m_r} \left[P - Q + \frac{2\xi_r}{\sqrt{1-\xi_r^2}} R \right] + \{A\}^T [H_{res})] \{V\} \tag{19}$$

With:

- $P = \sum_i \sum_j \chi_i \mu_j \Re(\Psi_{ri}) \Re(\Psi_{rj})$
- $Q = \sum_i \sum_j \chi_i \mu_j \Im(\Psi_{ri}) \Im(\Psi_{rj})$
- $R = \sum_i \sum_j \chi_i \mu_j \Re(\Psi_{ri}) \Im(\Psi_{rj})$
- ξ_r the viscous damping coefficient of the r^{th} mode

4 Experimental application to a car body

4.1 Dynamic measurements

The dynamic measurements are done in free-free condition on a body structure of a mass-produce vehicle as presented in figure 5.

Figure 5: Suspended car body for dynamic measurements

The elastic suspensions are tuned to have the resonance frequencies of the rigid body modes below 10% of the first torsion resonance frequency. A modal shaker is used for the excitation and accelerations are measured at twenty points using 3D accelerometers. A special impedance head is fixed between the shaker and the structure. The excitations are chosen in order to inject enough energy to measure in a good condition the first resonances of the structure. The measurement points are mainly located on the underbody as required by the two methods to determine the static stiffness of the car body. Additional points are used on the roof to help identifying the global modes of the structure. The mesh of the measurement points is shown in figure 6.

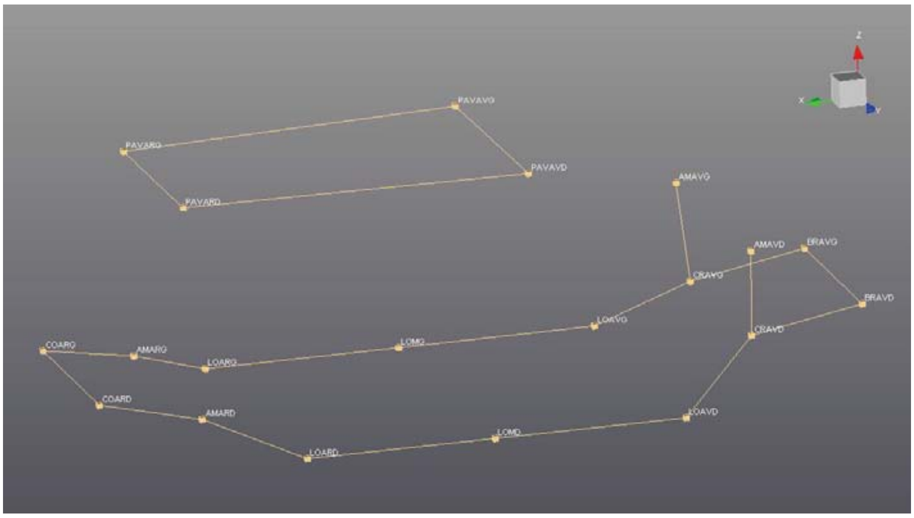

Figure 6: Mesh of the measurement points

4.2 Modal extraction

The standard best-practice of the modal analysis measurement should be followed for both the eRCF and the modal method:

- check the cross transfer functions symetry,

- comparison of the impedance at the excitation point between an impact hammer and the shaker to avoid modal effect of the link between the structure and the shaker,
- check the linearity of the measurement.

The PolyMAX modal extraction algorithm [7] is recommended to extract the modal basis of the car body, compared to the Time DOF algorithm. Only poles with physical meaning must be selected in the stabilization diagram to form the modal basis. An illustration of one stabilization diagram obtained is given in figure 7.

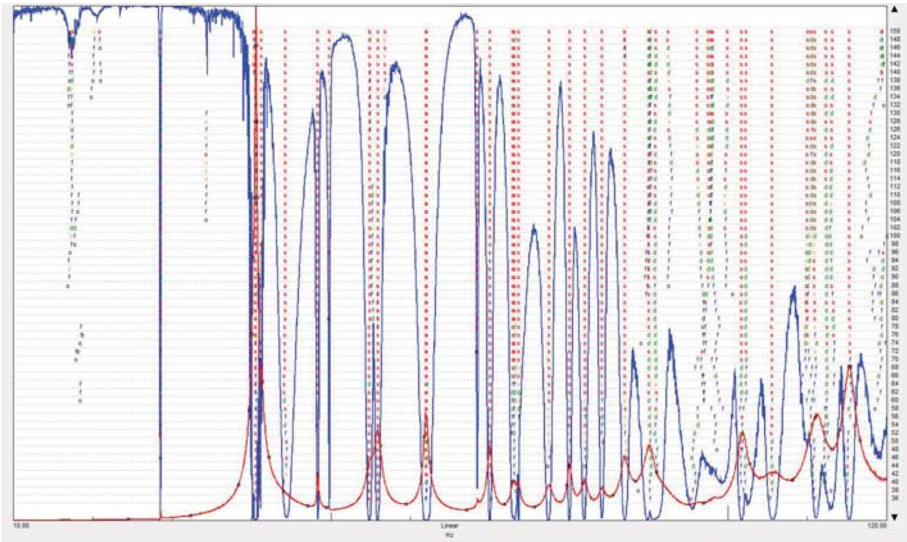

Figure 7: Stabilization diagram

According to the figure 8, the AutoMAC indicator points out that the 36 extracted modes are uncoupled enough.

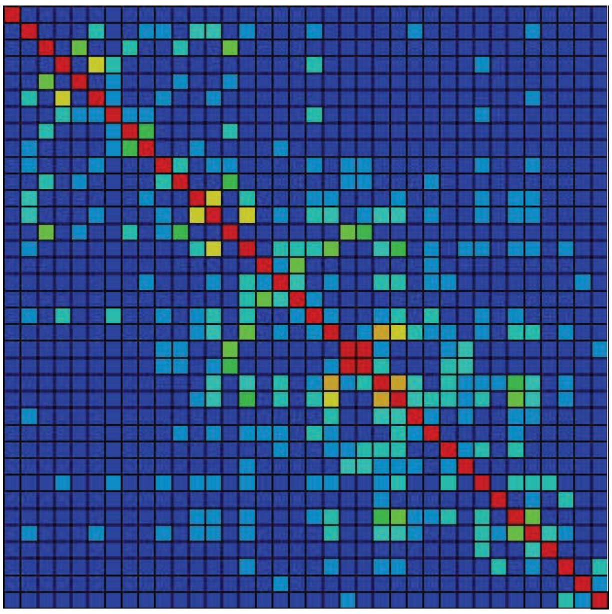

Figure 8: AutoMAC

4.3 Estimation of the static torsion angle

The modal basis is directly used in the modal approach to determine the static compliance. In the case of the eRCF, the modal basis is used to synthesize the FRFs and the resulted compliance matrix. The accuracy of the modal basis is a first order parameter in the determination of the static stiffness using the two methods. As presented in the Figure 9, a curve fitting of the eRCF is done in the low frequency domain in order to get the angle value at 0 Hz (equation 17).

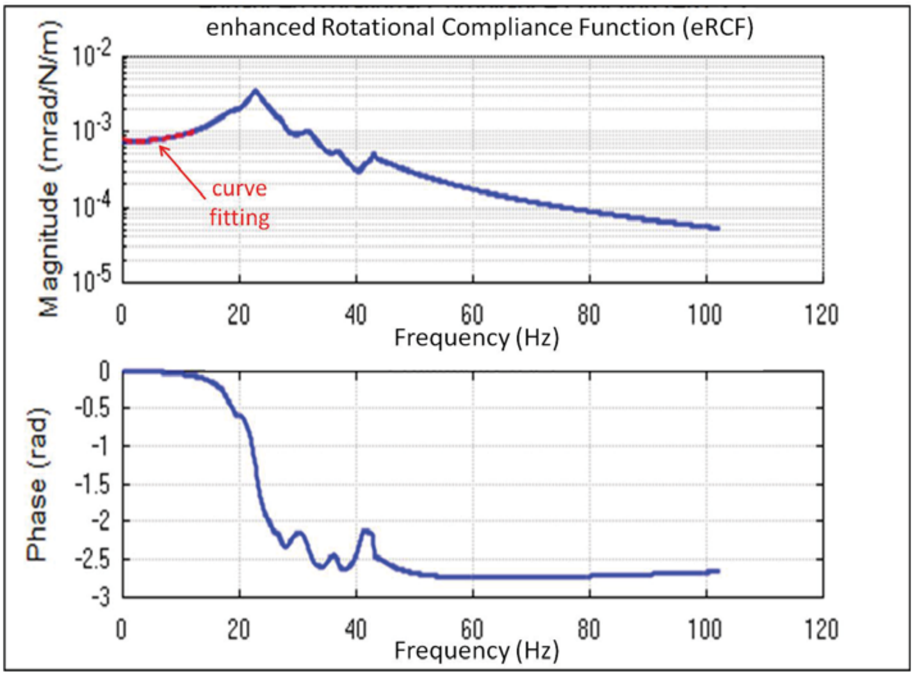

Figure 9: eRCF and curve fitting

The static angle values obtained with the two methods are compared to the reference PSA static test. Both are very close to the result of static test.

We can observe a good accuracy of the two methods with differencies of 1% for the modal method and 2% for the eRCF. The both of them slightly underestimate the torsional static stiffness of the car body. The modal method would lead to an angle of 0.86 mrad (error of 12%) without taking into account the residual terms.

Static test	eRCF	Modal method
0.98	1	0.99

Table 1: Calculation results vs Test measurements in mrad for a torque of 100 daN.m

5 Conclusion

Two methods to derive static stiffness from dynamic measurements are investigated in the case of the torsion of a car body. Their theories are developed to be able to compare the static estimates with reference values measured in a static test facility. An application case involving dynamic measurements of the car body of a mass-produce vehicle demonstrates a good accuracy of both methods with discrepancies up to 2%. Focusing on the modal approach, residual stiffness vectors are added in the equation used to determine the static compliance as a cumulative contribution of the modes of vibration in order to avoid modal truncation.

The analysis of the contribution of each mode on the static compliance is a way of perspective for car body engineering. A next step of the work will be the application of the two methods in the case of a full vehicle. As the structure of a full vehicle is highly damped because of trim components, the use of complex modes should be the key point of the modal approach.

Acknowledgements

The authors wish to thank Philippe Bieron, in charge of the PSA BIW Test Laboratory, for these explainations on static measure, Bruno Chamont, the low and mid frequency team manager of PSA Groupe, for providing financial support to this project, and the actors of the PSA PLM Masse for the financement.

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